Swaps

Bjørn Eraker

Wisconsin School of Business

January 16, 2015

Interest Rate Swaps

An interest rate swap is an agreement between two parties to exchange fixed for floating rate interest rate payments.

- The floating rate leg is typically pegged to LIBOR
- The fixed rate leg is determined at initiation

Fixed rate is determined in such a way that the market is indifferent between holding the floating/fixed rate leg of the contract. Thus, entering into the swap arrangement is *costless* at time 0.

Example

Table : 18.4 Two year cash flows on a 10 year, 5.688% fixed rate swap

date	libor	date	days	floating receipts	30/360	fixed payments
11/26/01	2.156	11/28/01	88			
02/26/02	2.000	02/28/02	92	550,883	90	
05/26/02	1.900	05/28/02	89	494,444	90	2,844,000
08/26/02	2.000	08/28/02	92	485,556	90	
11/26/02	2.100	11/28/02	93	516,667	91	2,859,800
02/26/03	2.200	02/28/03	91	530,833	89	
05/28/03	2.300	05/28/03	89	543,889	90	2,828,200
08/26/03	2.400	08/28/03	92	587,778	90	
		11/28/03	92	613,333	90	2,844,000

For example, the cash flow on 05/28/02 is based on the 02/26/02 LIBOR of 2%. With 100M notional amount, the fixed receives

$$100,000,000 \times \frac{2\% \times 89}{360} = 494,444$$

from the floating payer and pays

$$100,000,000 \times \frac{5.688\% \times 180}{360} = \$2,844,000$$

on 05/28/02. Here, the fixed rate payer receives some 500K every quarter and pays about 2.8 M every 6M for a combined average loss of almost 1.8M every 6m.

Valuation of swaps

Principle: Arrange payments such that the floating and fixed sides are indifferent.

Lets think of the swap as being a long + short position in two bonds:

- One fixed rate with semi-annual coupons
- One floating rate note with quarterly coupons where the floating rate pegged to the previous period Libor
- Make sure the value of both is par
- If so, both the principal and the par values cancel in the long-short portfolio

Valuation of the floating rate leg

Notice that the floating rate payments are determined based on *previous* period's libor rate.

Suppose the libor rate is $L(T-.25)$ one quarter prior to expiration of the swap at time T. Then at time T, we get paid

$$100(1+L(T-.25)\frac{90}{360})$$

at date T.

The present value of this payment is

$$\frac{100(1+L(T-.25)\frac{90}{360})}{(1+L(T-.25)\frac{90}{360})} = 100$$

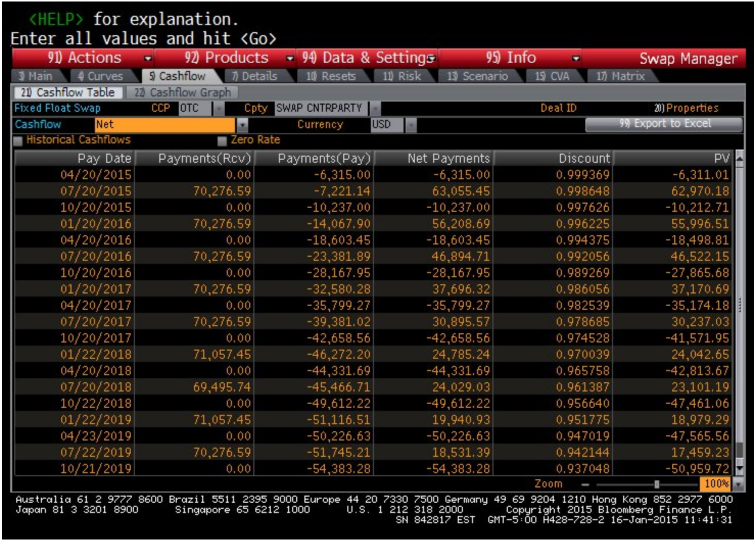
at time $T-0.25$ (one quarter before the final payment).

Since the discount factor is exactly the same as the interest payment, the floating rate note's value will always be par.

The combined cash flows from the fixed and long floating rate bonds are thus:

Table : Payments of fixed, floating rate notes and the difference (swap)

date	0	1/4	1/2	3/4	1	$\cdots$	T
fixed	-100	0	$\frac{c}{2}$	0	$\frac{c}{2}$	$\cdots$	$100+\frac{c}{2}$
floating	-100	$\frac{L(0)}{4}$	$\frac{L(\frac{1}{4})}{4}$	$\frac{L(\frac{1}{2})}{4}$	$\frac{L(\frac{3}{4})}{4}$	$\cdots$	$100+\frac{L(T-\frac{1}{4})}{4}$
fixed-float	0	$-\frac{L(0)}{4}$	$\frac{c}{2}-\frac{L(\frac{1}{4})}{4}$	$\frac{L(\frac{1}{2})}{4}$	$\frac{c}{2}-\frac{L(\frac{3}{4})}{4}$	$\cdots$	$\frac{c}{2}-\frac{L(T-\frac{1}{4})}{4}$



Pay Date	Payments(Rcv)	Payments(Pay)	Net Payments	Discount	PV
04/20/2015	0.00	-6,315.00	-6,315.00	0.999369	-6,311.01
07/20/2015	70,276.59	-7,221.14	63,055.45	0.998648	62,970.18
10/20/2015	0.00	-10,237.00	-10,237.00	0.997626	-10,212.71
01/20/2016	70,276.59	-14,067.90	56,208.69	0.996225	55,996.51
04/20/2016	0.00	-18,603.45	-18,603.45	0.994375	-18,498.81
07/20/2016	70,276.59	-23,381.89	46,894.71	0.992056	46,522.15
10/20/2016	0.00	-28,167.95	-28,167.95	0.989269	-27,865.68
01/20/2017	70,276.59	-32,580.28	37,696.32	0.986056	37,170.69
04/20/2017	0.00	-35,799.27	-35,799.27	0.982539	-35,174.18
07/20/2017	70,276.59	-39,381.02	30,895.57	0.978685	30,237.03
10/20/2017	0.00	-42,658.56	-42,658.56	0.974528	-41,571.95
01/22/2018	71,057.45	-46,272.20	24,785.24	0.970039	24,042.65
04/20/2018	0.00	-44,331.69	-44,331.69	0.965758	-42,813.67
07/20/2018	69,495.74	-45,466.71	24,029.03	0.961387	23,101.19
10/22/2018	0.00	-49,612.22	-49,612.22	0.956640	-47,461.06
01/22/2019	71,057.45	-51,116.51	19,940.93	0.951775	18,979.29
04/23/2019	0.00	-50,226.63	-50,226.63	0.947019	-47,565.56
07/22/2019	70,276.59	-51,745.21	18,531.39	0.942144	17,459.23
10/21/2019	0.00	-54,383.28	-54,383.28	0.937048	-50,959.72

Figure : SWPM in Bloomberg, Jan 16, 2015. Cash flows tab.

The SWAP rate is basically just the YTM/coupon rate of a bond with credit quality similar to Libor banks.

Therefore, we can define the SWAP spread to be the difference between swap and treasury rates of similar maturity.

Valuing the fixed rate leg

The fixed rate leg is just a bond selling at par!

Therefore: The question is not how to value the fixed rate leg, but rather how do we set up a coupon on the fixed rate leg which is such that the bond will sell exactly at par at inception.

Finding the swap curve from zero curve

For the fixed leg to be selling at par, the par yield on a T maturity, c_T must satisfy

$$100 = \sum_{i=1}^{2T} d(i/2)\frac{c}{2} + 100d(2T) \qquad (1)$$

where $d()$ is the discount function.
It is now easy to show that the par (swap) yield y_T must satisfy

$$c_T = 2 \times 100 \left(\frac{1 - d(2T)}{\sum_{i=1}^{2T} d(i)} \right) \qquad (2)$$

So if a T maturity, semi-annual Treasury has a coupon equal to c_T it will sell at par.

Example

A	B	C	D	E
		BASIC SWAP RATE ILLUSTRATION		
	zero ytm (%)	d(t)	Cash-Flows Fixed Rate leg	
0.5	0.2	0.9990005	1.31764515	
1	0.6	0.99401796	1.31764515	
1.5	1.1	0.98363538	1.31764515	
2	1.6	0.96850658	1.31764515	
2.5	2.1	0.94885432	1.31764515	
3	2.2	0.93613086	1.31764515	
3.5	2.4	0.91943126	1.31764515	
4	2.55	0.90302955	1.31764515	
4.5	2.6	0.88958519	1.31764515	
5	2.65	0.87590293	101.317645	
	5 year swap	2.6352903	100	

Figure : Excel basic Swap rate calculation.

The excel spreadsheet assumes a set of zero coupon yields, y_t, (continuously compounding) in column B. Column C computes the discount function $d(t) = \exp(-ty(t))$. The 5 year swap rate is then computed using (2). Column D gives the cash flows from the fixed rate leg and cell D13 verifies that the NPV is \$100.

SWAPS in Bloomberg

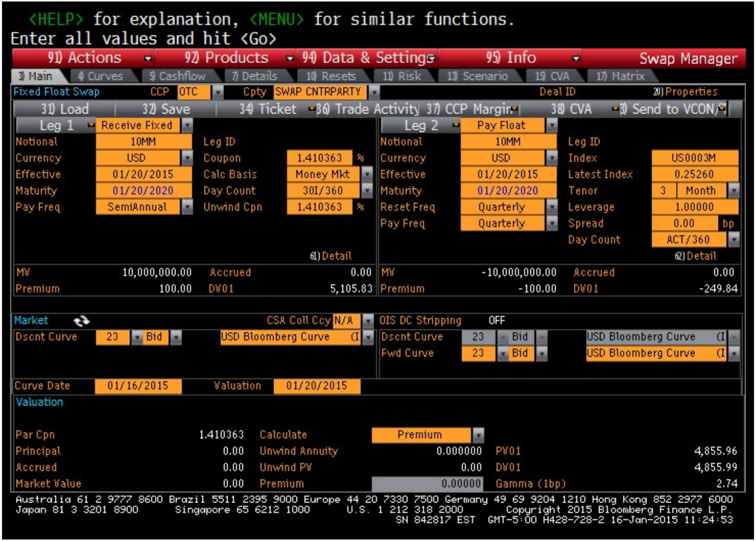


Figure : SWPM in Bloomberg, Jan 16, 2015

Note:

- The screen shows a 5 year swap
- The SWPM screen finds the swap rate (1.41 in this case) given default inputs
- We can change those inputs (try to change the floating rate Tenor to 1M, 6M, 12M.... what happens to the swap rate?)
- Notice that the DV01 on 10M are stated for both legs. It is 5,105.83 (fixed) and -249.84 (floating).
- Compare to Jan 20 maturity (1 3/8 coupon) selling at approximately par (see next page)

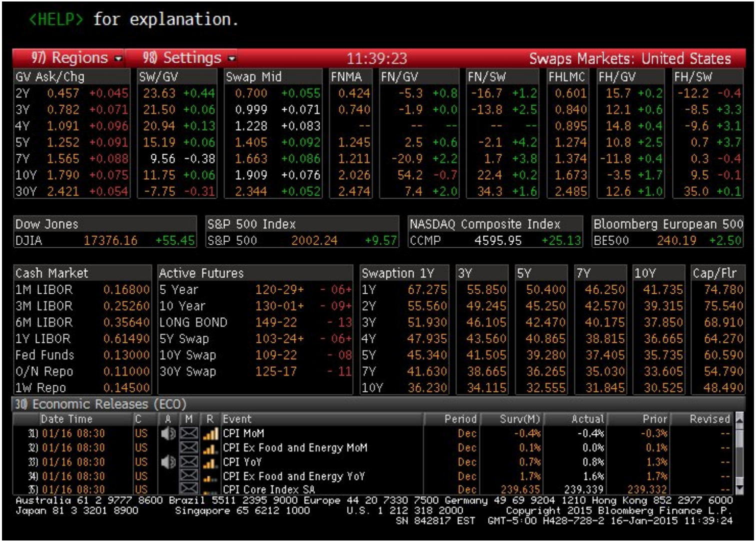
SWAPS in Bloomberg

Figure : 1 3/8 1/31/20 Maturity Treasury

The YTM on the Treasury is 1.2656 while the 5 yr swap rate is 1.41.

Why a spread?

There's actually a spread in most maturities. The USSW overview screen gives a spread for maturities up to 30 years.



<HELP> for explanation.

97) Regions ▾ 98) Settings ▾ 11:39:23 Swaps Markets: United States

	GV Ask/Chg		SW/GV		Swap Mid		FNMA	FN/GV		FN/SW		FHLMC	FH/GV		FH/SW	
2Y	0.457	+0.045	23.63	+0.44	0.700	+0.055	0.424	-5.3	+0.8	-16.7	+1.2	0.601	15.7	+0.2	-12.2	-0.4
3Y	0.782	+0.071	21.50	+0.06	0.999	+0.071	0.740	-1.9	+0.0	-13.8	+2.5	0.840	12.1	+0.6	-8.5	+3.3
4Y	1.091	+0.096	20.94	+0.13	1.228	+0.083	--	--	--	--	--	0.895	14.8	+0.4	-9.6	+3.1
5Y	1.252	+0.091	15.19	+0.06	1.405	+0.092	1.245	2.5	+0.6	-2.1	+4.2	1.274	10.8	+2.5	0.7	+3.7
7Y	1.565	+0.088	9.56	-0.38	1.663	+0.086	1.211	-20.9	+2.2	1.7	+3.8	1.374	-11.8	+0.4	0.3	-0.4
10Y	1.790	+0.075	11.75	+0.06	1.909	+0.076	2.026	54.2	-0.7	22.4	+0.2	1.673	-3.5	+1.7	9.5	-0.1
30Y	2.421	+0.054	-7.75	-0.31	2.344	+0.052	2.474	7.4	+2.0	34.3	+1.6	2.485	12.6	+1.0	35.0	+0.1

Dow Jones			S&P 500 Index			NASDAQ Composite Index			Bloomberg European 500		
DJIA	17376.16	+55.45	S&P 500	2002.24	+9.57	CCMP	4595.95	+25.13	BE500	240.19	+2.50

Cash Market		Active Futures		
1M LIBOR	0.16800	5 Year	120-29+	- 06+
3M LIBOR	0.25260	10 Year	130-01+	- 09+
6M LIBOR	0.35640	LONG BOND	149-22	- 13
1Y LIBOR	0.61490	5Y Swap	103-24+	- 06+
Fed Funds	0.13000	10Y Swap	109-22	- 08
O/N Repo	0.11000	30Y Swap	125-17	- 11
1W Repo	0.14500			

Swaption	1Y	3Y	5Y	7Y	10Y	Cap/Flr
1Y	67.275	55.850	50.400	46.250	41.735	74.780
2Y	55.560	49.245	45.250	42.570	39.315	75.540
3Y	51.930	46.105	42.470	40.175	37.850	68.910
4Y	47.935	43.560	40.865	38.815	36.665	64.270
5Y	45.340	41.505	39.280	37.405	35.735	60.590
7Y	41.630	38.665	36.265	35.030	33.605	54.790
10Y	36.230	34.115	32.555	31.845	30.525	48.490

30) Economic Releases (ECO)

Date Time	C	A	M	R	Event	Period	Surv(M)	Actual	Prior	Revised
31) 01/16 08:30	US				CPI MoM	Dec	-0.4%	-0.4%	-0.3%	--
32) 01/16 08:30	US				CPI Ex Food and Energy MoM	Dec	0.1%	0.0%	0.1%	--
33) 01/16 08:30	US				CPI YoY	Dec	0.7%	0.8%	1.3%	--
34) 01/16 08:30	US				CPI Ex Food and Energy YoY	Dec	1.7%	1.6%	1.7%	--
35) 01/16 08:30	US				CPI Core Index SA	Dec	239.635	239.339	239.332	--

Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2015 Bloomberg Finance L.P.
SN 842817 EST GMT-5:00 H428-728-2 16-Jan-2015 11:39:24

Figure : USSW Screen

The SW/GV column gives the swap spread over on-the-run treasuries. The 5 year spread is stated to be 15.19 BP but the

difference is 1.405 - 1.252 = 15.30 BP

For the 10 year it is 11.75 and the difference is 1.909-1.79 = 11.9 BP

Price Risks I

While swaps are basically designed to be buy-and-hold strategies, what happens if you opt out of the arrangement before the end of the contract?

You have to buy out the counter-party. Suppose we can do this for an exchange of cash. What is a fair exchange?

Suppose we receive fixed and pay floating. In this case, we are long a fixed rate coupon bond which market value is given as the present value of the fixed coupon and principal. We are short a floating rate bond that is always worth par.

Therefore: We must compensate the swap counterparty an amount equal to the difference between the current market value of the fixed leg (i.e., the coupon bond value), and par.

If interest rates increase, the fixed rate receiver in the swap contract has a capital (paper) loss. The fixed rate payer has a capital gain. And vise versa.

In other words, the fixed rate recipient has the same interest rate risk (i.e, duration) as that of a long bond holder. Swaps is a cheap way to bet on interest rate moves.

The example continued

Suppose we enter into the swap contract to receive fixed. Immediately we get a shift in the zero coupon curve as the long end goes up 5BP:

BASIC SWAP RATE ILLUSTRATION

	zero ytm (%)	d(t)	Cash-Flows Fixed Rate leg
0.5	0.2	0.9990005	1.31764515
1	0.6	0.99401796	1.31764515
1.5	1.1	0.98363538	1.31764515
2	1.6	0.96850658	1.31764515
2.5	2.1	0.94885432	1.31764515
3	2.2	0.93613086	1.31764515
3.5	2.4	0.91943126	1.31764515
4	2.6	0.9012253	1.31764515
4.5	2.65	0.88758588	1.31764515
5	2.7	0.87371591	101.317645
	5 year swap	2.6352903	99.7734043

We see that the value of the fixed rate leg is no longer 100.

Suppose we call the swap dealer and ask to exit the swap contract. We can do this, but it will cost us. How much?

$$100 - 99.7734 = 0.2266$$

So we have to compensate the swap counter party 0.2266 to exit the contract.

The important thing to remember about swaps:

By receiving fixed you are essentially buying a bond and financing that purchase by rolling it over at the LIBOR rate.

You can apply that insight to measure capital gains/ losses, duration, etc.

Balance sheet implications

A swap contract does not require an investment at time 0.

Therefore, it also does not appear on the balance sheets. Here's the implication: Suppose you artificially create a swap by purchasing a 10 year bond, and financing that purchase through short term borrowing at LIBOR.

The bond will appear on the asset side while the loan appears on the liability side of the balance sheet.

By contrast, a swap appears nowhere on the balance sheet!

Consider the following example: Suppose a bank has $100 in assets and $90 in debt. Consider two options:

A It enters a $10 notional swap to receive fixed

B It purchases $10 in treasuries, financing in whole by borrowing at libor

Note that the two are identical transactions in terms of cash flows and risk (disregard any swap spread).

However, in case A, the bank's balance sheet is unchanged and the leverage (debt to equity) is 9:1

In case B, its leverage goes from 9:1 to 10:1.

In some sense, the increased leverage in case B is reflective of the fact that the bank effectively increased its risk (duration) exposure. The balance sheet reflects this increase in risk.

In case A, the risks increase just as much as in case B, but there is no effect on the balance sheet.

The implication is that the derivative contracts (swap) increases the riskiness of the bank without changing the stated leverage. Therefore, regulation to curb leverage of banks is bound to fail if it focuses (only) on capital requirements in the form of traditional balance sheets, rather than more sophisticated measures of banks risk exposures.

Of course, this is also true for other zero-cost derivative contracts, especially credit default swaps.

Adding to both legs

Some swaps provide a spread over the libor. For example, the floating recipient gets LIBOR + X basis point. If so, we need to ad X basis points to the fixed rate leg as well. The swap then pays

$$L(t-0.25)/4+X/4-c/2-X/2$$

every 6 m and

$$L(t-0.25)/4+X/4$$

every quarter when the fixed leg is zero.

If both paid interest at the same time, the extra interest would cancel exactly. With quarterly and semi-annual payments, they approximately cancel...

Assume a flat swap curve at 6.1%.
Consider the following trade:

- buy 100M worth of FNMA 6.25 of May 15, 2029 at par
- Repo out the FNMA bond
- enter a swap to pay 6.25 (6.1+15 BP) to receive LIBOR+15 BP

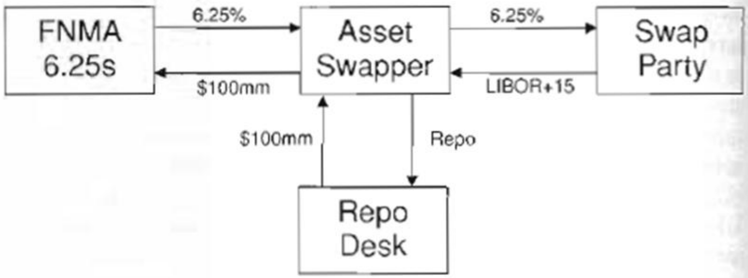


FIGURE 18.4 Asset Swap of the FNMA 6.25s of May 15, 2029

Net effect of this trade: Trader receives LIBOR+15 BP and pays repo. This is a good trade if the repo is below LIBOR + 15 BP.

Is it an arbitrage?

Price Risks II

Define the asset swap spread of the FNMA asset swap trade to be the difference between the LIBOR + 15 BP and the FNMA repo rate. Since both are short rates, there is no law of physics that will bound the repo rate to be below the LIBOR+15.

Clearly, if the asset swap spread becomes negative, the trades loses money. Figure 18.6 in T plots the spread and shows that the repo was substantially bellow the LIBOR+15 throughout 2000, but then went above LIBOR+15 in 2001. The asset swap trade thus lost money.

There are two possible explanations:

- The market considered the credit quality of FNMA to be worse than that of the average bank surveyed in the LIBOR pool.
- Liquidity in the FNMA bond was low so that trading desks were charging a steep repo rate (liquidity premium)

Price Risks III: Collateral calls

Lets consider a parallel shift in all the interest rates in the previous example. Consider a X basis points increase in both the FNMA ytm, the LIBOR, and the repo rates.

The rate increase has the following effects on the trade:

- Both interest income (from the LIBOR + 15 bp) and expense (from the repo loan) increase by X bp and thus cancel
- The value of the FNMA bond decreases.

The problem here is that if the value of the collateral (FNMA bond) in the repo trade decreases, the trader might face a margin or collateral call from his repo counter-party.

According to Tuckman, the combination of the reversal of the spread, decrease in the value of the FNMA bond, forced liquidation of the asset swap trades which led to further deterioration in the value of the FNMA bonds.

Counterparty default

Credit risk is not as severe of an issue as with risky debt per se because no principal ever changes hands.

Suppose, for the sake of argument, that the swap rates have not change since initiation of the the swap contract. In this case we stand to lose a maximum of one period interest payment.

In the case of an interest rate decrease (increase), the fixed recipient also loses (gains) an amount equal to the difference between market value of the fixed, and par.

Hedging with Swaps

Suppose an investor wishes to hedge a bond portfolio worth 10.2 M and with a modified duration of 8.35 with an interest rate swap.

Suppose further that the yield curve is flat at 5% and that he uses a plain vanilla swap with 10 year maturity an notional amount of 1 M. The swap's mod duration is 7.72 - the same as a 5% coupon selling at par (..remember that the interest rate sensitivity of a swap equals that of its fixed leg)

The hedge ratio is

$$\frac{\$dur_p}{\$Dur_S} = -\frac{10.2}{1} \times \frac{8.35}{7.72} \approx -11$$

so we should sell 11 swaps to immunize the portfolio.

CFA Notes

Reading 42 (3) and 50 are on Swaps.

Eqn. (12) in CFA 42.3 is the same as (1) here except that $s(T)$ (the swap rate), is defined in decimal and the spot rate curve is annually compounding. Our equation (1) uses semi-annual discounting, consistent with actual market practice.

You should be able to follow Ex. 8 in CFA 42.3, noting that all PV computations and cash flows are annual (equivalent to our example on slide 11).

The rest of section 42.3 is self-study, but should be straightforward.