Mortgage Backed Securities

Bjørn Eraker

Wisconsin School of Business

December 7, 2008

Outline

We will talk about *traditional* MBS's which include

- Mortgage pass through securities
- Collateralized mortgage obligations (CMO's)
- Stripped MBS's

These are securities issued by the Fannie, Freddie and Ginnie agencies.

Importantly: The US govt guarantees against default on the underlying mortgages in the traditional MBS's.

The govt does **not** guarantee the loans issued to subprime lenders in the 2000's which led to the massive 2008 crash and recession.

Two Types of Agency MBS's

Traditional, agency issued MBS's, therefore, are securities with risks relating to interest rates and mortgage repayments, but not default risk.

- Pass-through securities
- Collateralized Mortgage Obligations

Pass-through securities

Pass-through securities are pools of mortgages sold directly to investors. For example, FNMA sells the right to a specific pool of mortgage payments consisting of 1000 mortgage holders, averaging $200,000 in 200 units, each unit is entitled to 0.5% of the cash flows.

Thus, pass throughs give the investor direct ownership of a fraction of the mortgage pool.

CMO's

CMO's are divided into *trances* of securities with different seniority. For example, with three trances A,B,C, the A trance will receive principal repayments before B and C do.

The three tranches have the following characteristics:

- Tranche A absorbs the earliest principal pre-payments.
- Tranche B receives interest until tranche A is repaid in full. Tranche B then absorb the following principal pre-payments.
- Tranche C receives interest while A and B absorb pre-payments and is the last tranche to receive principal.

All three tranches has the characteristic of a short, medium and long maturity bond respectively.

Pre-payment risk

MBS's are fixed income securities which risk factors are

- Interest rate risk (in the same way as any fixed income security)
- Pre-payment risk.

Interest rates only partly determine the prepayments. Therefore, these are two distinct risk factors.

Factors determining prepayments:

- Refinancing (lower interest rates)
- Home sales (i.e., borrower moved, relocated to smaller, bigger house)
- Default
- Extra payments (homeowner wish to build home equity faster)
- Accidents (i.e., fire, flood, hurricanes)

A reasonable prepayment model incorporates *all* of the above.

Modeling Prepayments

Basic principle: A pool of mortgages is a *diversified* portfolio. The portfolio therefore will behave similarly to the national aggregates. For example, we can reasonably the actual number of home-sales in the pool to be equal to that of the national average turnover rate.

Two factors that determine valuation:

- term structure of interest rates
- expected repayments
- covariance between repayments and interest rates

Valuation

Assume that $C_t, t = 1, .., T$ are the cash flows (random) of the mortgage pool.
We will use the mother of all valuation formulas:

$$\text{Value} = E^* \left[\sum_{t=1}^{T} \frac{C_t}{\prod_{s=1}^{t} (1 + r_s)} \right] \quad (1)$$

where

- E^* denotes the risk neutral expectation.
- r_t is the spot rate

Note that the value *is not equal to the expected future payment discounted at the expected risk free rate*

$$\text{value} \neq \sum_t E(C_t)/(1 + E(r_t))^t.$$

We cannot factor this expectation operator through because

- doing so would require that $\text{corr}(C_t, r_t)$ is zero. But that is not true because the cash flows increase (increased prepayments) when interest rates decrease, and
- jensen's inequality

Mortgage valuation: take 1 - Trees.

Let's consider a model in which borrowers optimally execute the call imbedded in their mortgage.
This does not work well for two reasons:

- Borrowers don't execute optimally in that way because a number of other issues determine their prepayment behavior, and
- Standard interest rate trees don't work very well because the size of the asset pool depends on the entire history of interest rates, not just the current level. Modeling with trees would necessitate non-recombining trees.

So lets forget the option interpretation and trees altogether...

Mortgage valuation: take 2 - Monte Carlo

The most common way to value mortgage pools is to monte carlo simulate the possible future paths of interest rates, prepayments, etc.

Specifically, lets simulate

- $C_{t,j}$
- $r_{t,j}$

for N simulations, $j = 1, .., N$.
We now *estimate* the value as

$$\hat{\text{value}} = \frac{1}{N} \sum_{j=1}^{N} \sum_{t=1}^{T} \frac{C_{t,j}}{\prod_{s=1}^{t} (1 + r_{s,j})}$$

which will converge to its expected value as N increases.

Modeling cash flows and interest rates

Model ingredients:

1. We can use any reasonable model of interest rate behavior for simulation. In this class, we have previously simulated from CIR and Vasicek models. Of course, you can do something more advanced....
2. Next, we need to model pre-payments. It is particularly important to figure out how pre-payments tend to correlate with interest rate levels.
3. We need to model explicitly how mortgages amortize (mortgages are annuities). It is important to figure out what is left of the principal to get the size of a prepayment
4. How to deal with all factors affecting prepayments *unrelated to interest rate levels*

Briefly: Mortgage Math

Most fixed rate mortgages are annuities. An annuity loan for X dollars pay

$$c = X \left[\frac{\frac{R}{12}}{1 - (1 + \frac{R}{12})^{T \times 12}} \right] \tag{2}$$

per month. Each month, the amount

$$R_t = B_{t-1} \frac{R}{12}$$

is the interest rate payment, and

$$B_t = B_{t-1} - (c - R_t)$$

is the new balance.

A stylized excel example

We will consider a hypothetical mbs where the prepayments depend on the level of interest rates, and also on the age of the mortgage pool. The prepayments will occur more frequently in the beginning.

Specifically, I will assume a prepayment function

$$x_t = (\bar{x} + \kappa_x(r_t - \theta))\min(t/30, 1)$$

This prepayment function has the following features

- θ is average interest rate
- steady state, average prepayment is $\bar{x}$.
- dependence on interest rate levels incorporated through κ_x. When $\kappa_x < 0$ prepayments happen more frequently when interest rates are low.
- prepayments start out at zero, and then gradually increase. Here, they increase linearly for the first 30 months which is empirically reasonable.

Elements of the excel implementation

- Treat the pool as one mortgage. No need to model the individual mortgages.
- No need to model factors that are not correlated with interest rates. In other words, variation in prepayment that is *uncorrelated* with interest rates will not matter for the simulated average.
- Use VBA macro to do repeated simulations.

Results

Use WAC (weighted average coupon) = 6.5%, 30 year. This gives $c = 0.632$, or a monthly payment of 632 for a $ 100,000 loan.

Interest rate parameters are $\theta = 0.05, \kappa = 0.2, \sigma = .08$, and we assume $\kappa_x = -0.5$. This implies a max prepayment rate of 7.5%, close to its historical max.

I get the following results

Table: Effect of κ_x

wac=0.065		wac=0.055		wac=0.045	
κ_x		κ_x		κ_x	
0	120.29	0	112.69	0	101.50
-0.5	117.05	-0.5	108.06	-0.5	98.13

The price is always lower when $\kappa_x < 0$.

Interpretation: When borrowers refinance more quickly during low interest rate regimes the MBS loses value.