Duration

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Bjørn Eraker Duration

- Consider a 2.625% coupon bond with maturity 3/15/2009. Suppose the bond is selling at with settlement 9/23/2008 at a price 100.534 implying a yield (to maturity) of 1.634%.
- We are interested in what would happen to the value of this bond if interest rates change.
- Suppose the ytm changes from 1.634 to 1.734 (a 10 basis point change). What is the impact on the price?

The price at y = 1.734 is 100.487 - a -0.047 dollar change. Let $\triangle P$ and $\triangle y$ denote the change in the price and ytm, respectively.

We have $\triangle P = -0.047$ given $\triangle y = 0.1$. The relative change is

$$\frac{\triangle P}{\triangle y} = \frac{-0.047}{0.1} = -0.47$$

(the number is -0.4724 computed to four digits)

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Let's see what happens if y changes by only 5 basis points. Then the new price is 100.510 and

$$\triangle P = 100.510 - 100.534 = -0.024.$$

The relative change is

$$\frac{\triangle P}{\triangle y} = \frac{-0.024}{0.05} = -0.4725$$

Note that this number is close to the relative change when $\triangle y = 0.1$.

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Lets try even smaller $\triangle y$'s: We get

y(new)	Р	riangle P	riangle y	$\triangle P / \triangle y$
1.634001	100.534	0.000	1E-06	-0.4726
1.635	100.534		0.001	-0.4726

Note that as we change *y* very little, the ratio $\triangle P / \triangle y$ approaches the number -0.4726.

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This number is the *derivative* of the price as a function of the ytm at the point y = 1.634. The derivative is denoted

 $\frac{dP}{dy}$

We have found

 $\frac{dP}{dy} = -0.4726$

at y = 1.634. It is easy to demonstrate that a different price/ yield combination will imply a different derivative. For example if y = 5.634 then dP/dy = -0.4592.

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The risk measure "DV01" measures the sensitivity of a bond's price to a 0.01 change in the "interest rate," typically measured by the ytm of the bond.

DV01 gives the approximate absolute change in the value of the bond price in response to a 1 basis point change in the yield divided by 100,

$$DV01 = -\frac{\triangle P}{\triangle y}$$

with $\triangle y = 0.01$. I.e, a change from 5% to 5.01%.

The dollar duration of a bond is by definition

$$D_{\$} = -\frac{dP}{dy} \tag{1}$$

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where importantly y is the YTM measured in percent.

In our example $D_{\$}$ is just 0.4726.

To compute the dollar duration or DV01 in excel, we need a theoretical price (as function of YTM). The function "bondValue" in VB is included in the sheet "ytmBadDates2015.xlsm"

The arguments to "bondvalue" are

- coupon
- settlement date (excel date)
- expiration date (excel date)
- discount rate

When computed with discount rate = YTM, the value should by definition be equal to the market price.

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8.125	5 2/15/2015	912810ED6	129.375	1.5237289	1.5312657	1.5161948	129.375	132.4660326	132.460	
8.5	5 2/15/2015	912810EE4	133.75	1.6125932	1.6193003	1.6058884	133.75	136.9836957	136.963	
8.75	5 5/15/2015	912810EF1	136.375	1.6491996	1.6568086	1.6415937	136.375	137.535221	137.524	
8.7	5 2/15/2015	912810EG9	137.5469	1.7199346	1.7271947	1.7126774	137.546875	140.8756793	140.855	
7.87	5 2/15/2015	912810EH7	135.1406	1.7925002	1.7981579	1.7868445	135.140625	138.1365489	138.117	
8.12	5 5/15/2015	912810EJ3	137.625	1.841574	1.8469701	1.8361798	137.625	138.7023481	138.680	
8.12	5 2/15/2015	912810EK0	138.9844	1.8480422	1.8542493	1.8418376	138.984375	142.0754076	142.057	
7 21	5/15/2015	912810EL8	139.1875	1.8932843	1.8982845	1.8882857	139.1875	140.2482735	140.227	
7.62	5 2/15/2015	912810ENIA	141 2656	1.9256534	1.93114//	1.9201613	137.609375	140.30/52/2	140.353	
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Figure : Calling "bondValue" in "ytmBadDatesXXXX"

Bjørn Eraker

Duration

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We can now compute the DV01 or dollar duration numerically by increasing the YTM by one basis point and dividing by 0.01 (multiply by 100):

In the excel spreadsheet we do this by calling "bondvalue" again using a single basis point higher YTM, take the difference, and divide by 0.01 (see next page)

Here we have the baseline bond value in column L. Then we call the "bondValue" again with a higher YTM. The result is in column P.

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3	138.0938	2.4986016	2.4993532	2.4978501	138.09375	139.9008152	139.847	2.495	0.053507	0.003	20.785		
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	130.9375	2.5890077	2.5897672	2.5882483	130.9375	132.6019022	132.552	2.586	0.04942	0.002	20.566		
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	139,7813	2.6347235	2.6357234	2.6337238	139,78125	141.5883152	141.531	2,631	0.056885	0.002	23,431		
;	132.6719	2.646853	2.647884	2.6458223	132.671875	133.2519855	133.202	2.644	0.050132	0.002	22.725		
5	120.5938	2.6658524	2.666947	2.6647581	120.59375	122.0203804	121.974	2.663	0.046224	0.002	21.403		
	107.9531	2.707197	2.7079803	2.7064139	107.953125	108.3674896	108.319	2.704	0.048812	0.002	19.959		
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Figure : Numerical dollar duration compution "ytmBadDatesXXXX"

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1) Yield & Spread 2) Yields 3) Gra	aphs 4) Pricing	5) Descriptive	6) Custom	
T 4 3 05/15/41 (912810004)		Risk		
Price 132-21+ (132.671875	5)	Duration		17.293
Settle 01/02/15 🔳	Maturity	Modified Duration		17.067
	05/15/41	Risk		22.742
Street Convention	2.645822	Convexity		3.826
US Government Equivalent	2.645724	DV 🔹 01 on 1MM		2,274
True Yield	2.645716	YV 🔹 0.031		0.00137
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Mmkt (Act/ 360 -)		Principal		1,326,718.75
Current Yield	3.298	Accrued (48 Days)		5,801.10
		Total (USD)		1,332,519.85
After Tax (Inc 43.400 % CG 23.800 %) 1.497570			
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Figure : YA for the May 41 maturity

Bjørn Eraker

Duration

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The "Risk" number in BB is the dollar duration/ DV01.

- BB gives 22.742
- We got 22.72
- If we compute it using $\triangle Y = 0.001$ we get 22.748

There are also slight differences in the way BB computes the present value vs. our spreadsheet.

Remember that the value of the treasury with semi-annual coupons is

$$P = \sum_{i=1}^{n} \frac{C_i}{(1+y/2)^{\frac{2t_i}{365}}}.$$
 (2)

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(equation 4 in the "bondBasics" lecture).

We want to compute the derivative

 $\frac{dP}{dy}$

analytically.

We get

$$\frac{dP}{dy} = \sum_{i=1}^{n} \frac{d}{dy} C_{i} (1 + y/2)^{-\frac{2t_{i}}{365}}$$
(3)
$$= -\frac{1}{2} \sum_{i=1}^{n} \frac{2t_{i}}{365} C_{i} (1 + y/2)^{-\frac{2t_{i}}{365}-1}$$
(4)
$$= -\frac{1}{(1 + y/2)} \sum_{i=1}^{n} \frac{t_{i}}{365} \frac{C_{i}}{(1 + y/2)^{-\frac{2t_{i}}{365}}}$$
(5)
$$= -\frac{1}{(1 + y/2)} \sum_{i=1}^{n} t_{i}^{a} PV_{i}$$
(6)

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where

$$t_i^a = t_i/365 \tag{7}$$

is the time to cash-flow *i* in years, and

$$PV_i = \frac{C_i}{(1+y/2)^{-\frac{2t_i}{365}}}$$
 (8)

is the present value of cash flow i. Note that we can compute the dollar duration directly in VB if we please using equations (6) - (8).

```
Function dollarDuration(cpn, settle, expiration, y)
    nD = 182.5
    n = WorksheetFunction.CoupNum(settle, expiration, 2)
    firstCPNd = WorksheetFunction.CoupNcd(settle, expiration,
    s = firstCPNd - settle
    D = cpn / 2 * (1 + y / 2) ^ (-s / nD) * (s / nD)
    i = 1
    Do While i < n
        nextCPNd = nCPNdate(firstCPNd, i)
        s = nextCPNd - settle
        D = D + cpn / 2 * (1 + y / 2) ^ (-s / nD) * (s / nD)
        i = i + 1
    Loop
    D = D + 100 * (1 + v / 2) ^ (-s / nD) * (s / nD)
    dollarDuration = 0.5 \times D \times 100 / (1 + v / 2)
End Function
```

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When calling the function in excel we get 227,420.

What's wrong?

The number is obviously scaled by 10,000 relative to the previous "correct" number of 22.74.

What happened?

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Remember that we defined dollar duration to be the derivative wrt YTM measured in PERCENT. Equation (2) is a present value formula in which of course the discount rate is measured as a decimal number.

We thus need to scale the duration by 10000 to get the two numerical computation to agree. We do this by adding the code

```
dollarDuration = dollarDuration / 10000
```

to the end of the VB function.

Define D to be the Dollar Duration. The Modified Duration is defined

$$D_M = \frac{\$D}{P} \tag{9}$$

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in most textbooks. *P* is the price of the bond.

It's easy to verify that dividing the dollar duration ("RISK") for the 4.75 May 41 Treasury note gives the wrong number. Using the BB numbers we get

$$\frac{22.742}{133.25} = 0.1707$$

which is obviously scaled by 1/100.

So the correct formula, given our definition of dollar duration, is

$$D_M = \$ D \frac{100}{P} \tag{10}$$

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which gives 17.067.



is defined

$$D = (1 + y/2)D_M$$
 (11)

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Using the BB numbers again we get

$$D = 17.067 \times (1 + 0.026458/2) = 17.293$$

Suppose we have some position in a bond (or portfolio of bonds). For the sake of argument, suppose we guy 1M face value of the Nov 41, 4 3/8 maturity.

We wish to take a second position to offset potential interest rate moves. We can accomplish this by selling short a certain amount of a different bond. Let's consider the May 41 maturity.

On the next two pages you find the Bloomberg YA screen for these two bonds on Dec 31st, 2014.

v	Λ
Y	н

T 4 3 05/15/41 Govt			Yield and	Spread Analysis
132-20/132-21+ 2.648/2.646	BGN @ 14:0	0 95) Buy	96) Sell	97) Settings 📼
1) Yield & Spread 2) Yields 3) G	raphs 4 Pricing	5) Descriptive	6) Custom	
T 4 3 05/15/41 (912810QQ4)		Risk		
Price 132-21+ (132.67187	75)	Duration		17.293
Settle 01/02/15	Maturity	Modified Duration		17.067
	05/15/41	Risk		22.742
Street Convention	2.645822	Convexity		3.826
US Government Equivalent	2.645724	DV 🔹 01 on 1MM		2,274
True Yield	2.645716	YV 🔹 0.031		0.00137
Equiv 1 🔄 /Yr Compound	2.663323	Invoice		
Japanese Yield (Simple)	2.363000	Face		1,000 M
Mmkt (Act/ 360 🖬)	Second Second	Principal		1,326,718.75
Current Yield	3.298	Accrued (48 Days)		5,801.10
		Total (USD)		1,332,519.85
After Tax (Inc 43.400 % CG 23.800	%) 1.497570			
Issue Price = 102.986. Bond Purchased with Pre-	emium.			
Australia 61 2 9777 8600 Brazil 5511 239 Japan 81 3 3201 8900 Singapore 65 6	5 9000 Europe 44 20 212 1000 U.S.	7330 7500 Germany 49 1 212 318 2000 C	69 9204 1210 Hong opyright 2014 Blo	Kong 852 2977 6000 omberg Finance L.P.
		SN 842817 EST GMT-5	00 H444-4579-2 3:	1-Dec-2014 17:18:45

Figure : YA for the May 41 maturity on Dec 31st, 2014

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Duration

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T 3 ¹ 8 11/15/41 Govt			Yield and	Spread Analysis
107-29+/107-30+ 2.708/2.706	BGN @ 14:0	95) Buy	96) Sell	97)Settings 📼
1) Yield & Spread 2) Yields 3) Gra	aphs 4) Pricing	5) Descriptive	6) Custom	
T 3 ¹ s 11/15/41 (912810QT8)		Risk		
Price 107-30+ (107.953125	5)	Duration		18.662
Settle 01/02/15 🔳	Maturity	Modified Duration		18.413
	11/15/41	Risk		19.954
Street Convention	2.706414	Convexity		4.331
Treasury Convention	2.706318	DV 🗾 01 on 1MM		1,995
True Yield	2.706326	YV 🔳 0.031		0.00157
Equiv 1 🔄 /Yr Compound	2.724726	Invoice		
Japanese Yield (Simple)	2.620000	Face		1,000 M
Mmkt (Act/ 360 🖬)	Sector Sector Sector	Principal		1,079,531.25
Current Yield	2.895	Accrued (48 Days)		4,143.65
		Total (USD)		1,083,674.90
After Tax (Inc 43.400 % CG 23.800 %) 1.531857			
Issue Price = 102.747. Bond Purchased with Pren	nium.			
Australia 61 2 9777 8600 Brazil 5511 2395 Japan 81 3 3201 8900 Singapore 65 62	9000 Europe 44 20 12 1000 U.S.	1 2330 2500 Germany 49 1 212 318 2000 C	69 9204 1210 Hong opyright 2014 Blo	Kong 852 2977 6000 omberg Finance L.P.
		SN 842817 EST GMT-5	00 H444-4579-3 3	1-Dec-2014 16:00:53

Figure : YA for the Nov 41 maturity on Dec 31st, 2014

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Let's just sell short 1M worth of face of the Nov maturity to see what happens.

We see that

- We pay \$1,332,519 on Dec 31st for 1M face of the May maturity.
- We receive \$1,083,674 in cash for shorting the Nov maturity.

Now let's see what happened to our position on Jan 2d, 2015:

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T 4 3 05/15/41 Govt			Yield and !	Spread Analysis
134-03/134-04+ 2.583/2.581	BGN @ 16:	24 95) Buy	96) Sell	97)Settings 📼
1) Yield & Spread 2) Yields 3) Gra	phs 4) Pricin	g 5 Descriptive 6) Custom	
T 4 🖁 05/15/41 (912810QQ4)		Risk		
Price 134-04+ (134.140625)	Duration		17.339
Settle 01/05/15 🔳	Maturity	Modified Duration		17.118
	05/15/41	Risk		23.067
Street Convention	2.581373	Convexity		3.842
US Government Equivalent	2.581276	DV 01 on 1MM		2,307
True Yield	2.581269	YV 🔳 0.031		0.00135
Equiv 1 / Yr Compound	2.598032	Invoice		
Japanese Yield (Simple)	2.295000	Face		1,000 M
Mmkt (Act/ 360 🖬)		Principal		1,341,406.25
Current Yield	3.262	Accrued (51 Days)		6,163.67
		Total (USD)	8	1,347,569.92
After Tax (Inc 43.400 % CG 23.800 %)	1.461092			
Issue Price = 102.986. Bond Purchased with Prem	ium.			

Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. I 212 318 2000 Courtight 2015 Bloomberg Finance L.P. N 842817 EST GMT-5100 Hi87-3892-6 02-40n-2015 16-24:52

Figure : YA for the May 41 maturity on Jan 2d, 2015

Bjørn Eraker

Duration

T 3 🔓 11/15/41 Govt			Yield and	Spread Analysis
109-06+/109-07+ 2.644/2.643	BGN @ 16:	22 95) Buy	96) Sell	97)Settings 📼
1) Yield & Spread 2) Yields 3) Gra	phs 4) Pricing	g 5) Descriptive 6	Custom	
T 3 🔓 11/15/41 (912810QT8)		Risk		
Price 109-07+ (109.234375)	Duration		18.709
Settle 01/05/15 🔳	Maturity	Modified Duration		18.465
	11/15/41	Risk		20.251
Street Convention	2.642583	Convexity		4.348
Treasury Convention	2.642489	DV 01 on 1MM		2,025
True Yield	2.642497	YV 🔳 0.031		0.00154
Equiv 1 /Yr Compound	2.660041	Invoice		
Japanese Yield (Simple)	2.546000	Face		1,000 M
Mmkt (Act/ 360 🖬)		Principal		1,092,343.75
Current Yield	2.861	Accrued (51 Days)		4,402.62
		Total (USD)	÷.	1,096,746.37
After Tax (Inc 43.400 % CG 23.800 %)	1.495729			
Issue Price = 102.747. Bond Purchased with Prem	ium.			

Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. I 212 318 2000 Courtight 2015 Bloomberg Finance L.P. N 842817 EST GMT-5100 Hi87-3892-6 02-400-2015 Hc23+45

Figure : YA for the Nov 41 maturity on Jan 2d, 2015

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We now unload the position at market prices. We get

• 1,347,569 for the May maturity

• We pay 1,096,746 to cover the short on the Nov maturity In total we gained 1,347,569 - 1,332,519 = 15,050 on the long position and -1,096,746 + 1,083,674 = -13,072 on the short.

The hedge almost works and we get a gain of about 2K. This ideally should be zero. Let's see if we can do better....

What messed up our hedge is that the DV01 on 1M of the May is higher than the DV01 on the Nov. (2,307 vs 1,995).

Meanwhile, the YTM on the two bonds went from 2.646 to 2.581 (down 6.5 BP) and 2.706 to 2.643 (down 6.3 BP), respectively.

Let F_a and F_b denote the face amount of each bond. The following is the amount of face of *a* we need to hedge F_b :

$$F_a = \frac{-F_b DV01_b}{DV01_a} \tag{12}$$

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So in our example,

$$F_a = -1M \frac{2274}{1995} = -1.1398M \tag{13}$$

By doing 1.1398 M of face of the Nov maturity we get a capital gain of

 $-1.1398 \times (1,096,746 - 1,083,674) = -14,900.$

Remember that the gain on the May maturity was 15,050 so wet are off by only \$150.

Moral of the story: The duration hedge works (almost perfectly).

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Consider the BB screens that follow on the next two pages.

Notice that for the 30 year bonds, the yields went down about 6-7 BP.

For the 1-2 year notes, yields did not change...

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United States	1)	Actions	s 🔹 💈) Tools		3) Settings	 Fixed 	Income	e Trading
17:47 * Market Closed *									
4) Actives	5) Bills) Notes) TIPS		8) Strips	9) Sp	rds	10) Ci
21) T/0-1 22)	T/1-2 23) T/2-4 24	T/4-7	25) T/7	-10 26) T/10	0-30			
	BidPx /AskPx	AskYld	YlChg 💌			BidPx	/AskPx	AskY	ld <mark>YlChg</mark>
31) 0 ¹ ₄ D15	99-31 ³ ₄ / 100-00 ¹ ₈	0.246		49) 2 🖥	416	102-27°s/	102-28 ³ 4	0.417	+0.004
32) 2 ¹ 8 D15	101-25+ / 101-25 ⁷ 8	0.287	-0.007	50) 0 🖥	416	99-30 /	99-30 ⁵ s	0.408	+0.015
33) 0¹₄ D15	99-31 ¹ ₄ / 99-31 ³ ₄	0.258		51) 0 ¹ 4	516	99-24³ /	99-24 ⁷ 8	0.414	+0.004
34) 0 ³ 116	$100-02^{3}_{8} / 100-02^{3}_{4}$	0.291	+0.003	52) 5 ¹ 8	516	106-11 /	106-12 ¹ s	0.413	+0.019
35) 2 116	101-24+ / 101-25 ¹ 8	0.328	-0.002	53) 7 ¹ 4	516	109-05 /	109-08 ¹ 4	0.412	+0.010
36) 0 ³ 116	100-01 ⁷ / 100-02 ¹	0.309	-0.004	54) 314	516	103-285/	103-29 ³ 4	0.434	-0.002
37) 4 ¹ ₂ 216	104-18 ¹ / 104-19	0.355	+0.015	55) 134	516	101-2514 /	101-26 ¹ 8	0.448	+0.006
38) 9 ¹ ₄ 216	109-28 ¹ ₈ / 109-29 ⁵ ₈	0.297	-0.012	56) 0 🔒	516	99-29+ /	99-30 ¹ 4	0.414	+0.003
39) 0 ³ 216	100-01 / 100-01+	0.333	-0.011	57) 0 ¹ 2	616	100-02 ¹ s/	100-02 ⁵ 8	0.443	+0.005
40) 25 216	102-18 ⁵ / 102-19 ³	0.351	-0.006	58) 0 ¹ 2	616	100-00 🖥 /	100-01 ¹ 4	0.474	+0.005
41) 2 ¹ 8 216	102-00 ⁵ / 102-01 ¹ / ₄	0.345	-0.003	59) 3 ¹ 4	616	104-02 ¹ 8/	104-03+	0.472	-0.010
42) 0 ¹ ₄ 216	99-27+ / 99-28	0.359	-0.006	60) 1 ¹ 2	616	101-15+ /	101-16 ¹ 4	0.481	+0.002
43) 0 🖁 316	99-31 ⁷ s / 100-00 ¹ 4	0.368	+0.006	61) 3 ¹ 4	716	104-08+ /	104-0934	0.495	-0.007
44) 0³ ∎ 316	99-31 ³ / 99-31 ⁷ s	0.378	-0.006	62) 1 ¹ 2	716	101-16 🖥 🖊	101-17+	0.510	
45) 2 ³ 316	102-14 / 102-15	0.367	-0.007	63) 0⁵ <mark>8</mark>	716	100-0614 /	100-0634	0.486	
46) 2 ¹ ₄ 316	102-09+ / 102-1014	0.363	-0.006	64) 0 ¹ 2	716	100-00 ⁷ s /	100-01 ³	0.472	+0.002
47) 0 ¹ ₄ 416	99-26 / 99-27	0.373	+0.004	65) 4 ⁷ 8	816	106-30 % /	106-315	0.514	+0.007
48) 2 416	102-01 ³ / 102-02 ¹	0.423	+0.002	66) 0 5	816	100-04 ³ /	100-05	0.527	+0.002
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2377 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Courjght 2015 Bloomberg Finance L.P. N 842817 EST CMT-5100 H187-3392-20 2-147:30									

Figure : 1-2 year notes on Jan 2, 2015

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Duration

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United :	States		v 1)	Action	s = 2) Tools		3) Settings -	Fixed	Income	e Trading
17:46 * Market Closed *											
< 4) Ac	ctives	5) Bills		6) Notes		7) TIPS		8) Strips	9) Sp	9) Sprds	
21) T/O)-1 22) T/1-2 23) T	/2-4 24	T/4-7	25) T/7	-10 2) T/10	0-30			
		BidPx /A	\skPx	AskYld	YlChg -			BidPx /	AskPx	AskYl	d MChg
31) 234	N23	105-22 /	105-23	2.041	-0.059	59) 3 ⁷ 8	840	123-15 /	123-16+	2.610	-0.065
32) 2%	824	102-10+/	102-11	2.104	-0.061	60) 4 ¹ 4	N40	130-28+/	130-30	2.599	-0.065
33) 7 ¹ 2	N24	148-13+/	148-15+	2.045	-0.060	61) 4 ³ 4	241	141-08+/	141-10	2.569	-0.065
34) 75	225	150-14+/	150-16+	2.064	-0.060	62) 4 ³ 8	541	134-05 /	134-06+	2.579	-0.067
35) 67 ₈	825	145-06 /	145-07+	2.098	-0.061	63) 3 ³ 4	841	121-30+/	122-00	2.599	-0.065
36) 6	226	137-25+/	137-27	2.152	-0.061	64) 3 ¹ 8	N41	109-07+/	109-09	2.640	-0.066
37) 634	826	146-23 /	146-25	2.170	-0.063	65) 3 ¹ 8	242	108-30 /	108-31+	2.658	-0.064
38) 612	N26	144-23+/	144-25+	2.190	-0.062	66) 3	542	106-13+/	106-14+	2.666	-0.067
39) 65	227	146-24+/	146-26+	2.199	-0.063	67) 234	842	101-07 /	101-08	2.686	-0.065
40) 63	827	145-12+/	145-14	2.223	-0.064	68) 2 ³ 4	N42	101-05 /	101-06	2.689	-0.066
41) 6 ¹ 8	N27	143-06 /	143-08	2.236	-0.065	69) 31 ₈	243	108-21+/	108-22+	2.682	-0.065
42) 5 ¹ 2	828	137-16+/	137-18	2.275	-0.063	70) 278	543	103-17+/	103-19	2.693	-0.065
43) 5 ¹ 4	N28	135-03 /	135-05	2.277	-0.066	71) 35	843	119-00 /	119-01+	2.669	-0.065
44) 5 ¹ ₄	229	135-15+/	135-17	2.289	-0.066	72) 34	N43	121-20 /	121-21+	2.668	-0.064
45) 6 ¹ 8	829	147-10 /	147-12	2.291	-0.067	73) 3 5	244	119-03 /	119-04+	2.674	-0.064
46) 6 ¹ / ₄	530	150-18+/	150-20	2.314	-0.068	74) 3%	544	113-30+/	113-31+	2.684	-0.064
47) 5%	231	140-18+/	140-19+	2.336	-0.068	75) 31 ₈	844	108-29 /	108-30	2.685	-0.063
48) 4 ¹ 2	236	134-27+/	134-29	2.386	-0.070	76) 3	N44	106-11+/	106-12	2.688	-0.064

Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2015 Bloomberg Finance L.P. SN 842817 EST GMT-5:00 H187-3892-2 02-Jan-2015 17:46:28

Figure : 30 year notes on Jan 2, 2015

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This tells you that if we had tried to hedge the position in the May 41 with a position in any of the 1-2 year bonds, the hedge would have been useless.

This is so because the short end of the yield curve did not move. Only the long end did.

Consequently:

- Duration is a useful measure of risk only if you believe that the yield curve shifts in parallel all the time (not true)
- To hedge a single bond position, you need a bond (or other instrument) with very similar characteristics (i.e, maturity).
 I.e, you need "apples to hedge apples"
- It is rather useless to think of duration as representing an accurate measure of interest rate risk. Generally, the short end of the curve moves around a lot more than the long end, meaning that duration overstates the risk of long-maturity bonds relative to short maturity bonds,
- Even if two portfolios have the same duration, they may exhibit different interest rate sensitivity

The idea behind key rate duration is that, as we have seen, different segments of the yield curve will move around independently.

Key Rate Duration seeks to measure the sensitivity of a portfolio with respect to a local shift in the yield curve. This is most often done by measuring the sensitivity of a bond with respect to a change in a local segment of *the zero-coupon yield curve*.

Check out this youtube video for a stylized simplied example: https://www.youtube.com/watch?v=nQ4nbF0rfUA Note that what he refers to as the spot rate curve is the zero-coupon yield curve (continuously compounding).

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Let's try to understand (mathematically) why duration hedging work

The relationship between the dollar duration, the change in the yield, and the change in the bond price can be expressed approximately as

$$\triangle P \approx -\$ D \triangle y \tag{14}$$

We can think of $\triangle P$ as representing the change in the (dirty) bond price from day *t* to *t* + 1.

Suppose we have a portfolio with n_i number of bond *i* (so *n*100 is the face amount of bond *i*). The portfolio value is

$$V = \sum n_i P_i \tag{15}$$

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We then have that the change in the portfolio value is

$$dV = \sum_{i} n_i \triangle P_i = -\sum_{i} \$ D_i dy_i$$
(16)

To derive the hedging formula (12), assume that the yield on bond *a* change by as much as the yield on bond *b* $(dy_a = dy_b = dy)$. We need

$$0 = n_a \$ D_a dy + n_b \$ D_b dy$$

which gives

$$n_a = -n_b \frac{\$D_b}{\$D_a} \tag{17}$$

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or equation (12).

Equation (14) is an example of a First Order Taylor expansion. You can guess that there exists a second (and third, and....) Taylor expansion which is more accurate:

$$\triangle P \approx \frac{dP}{dy} \triangle y + \frac{1}{2} \frac{d^2 P}{dy^2} \triangle y^2$$
(18)

where $\frac{d^2 P}{dy^2}$ is the second order derivative of the price wrt the YTM.

The convexity of a bond is usually defined as

$$C = \frac{1}{P} \frac{d^2 P}{dy^2} \tag{19}$$

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Note that since the second order derivative is the same as the derivative of the first order derivative, we can think about convexity as measuring how much the durations change as the yield changes.

The convexities of the May and Nov 41 Notes are stated on the BB YA screens to be 3.826 and 4.331 respectively.

Let's consider constructing a convexity neutral portfolio. We need to solve

$$0 = n_a C_a + n_b C_b \tag{20}$$

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in addition to the equation for the duration neutrality.

We now have two equations but only one unknown, n_b , which we will therefore not be able to solve. We would need one additional bond to construct a convexity and duration neutral portfolio.