Credit Risk Modeling

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Background

- We usually assume default free bonds
- Until recently we thought the US govt could never default. They can.
- Other sovereign debt carry higher default probability
- Most relevant : Corporate bonds

Risks of investing in Corporate Bonds

The possibility of *default* ads a dimension of complexity. The two main concerns are

- The *likelihood* of default
- The *recovery rate* in the case of default

Corporate bonds *ytm's* are higher due to the possibility of default.

Insolvency

Two types of insolvency:

- Cash flow insolvency
  - Unable to pay debts as it comes due.
- Balance sheet insolvency
  - Negative equity

Insolvency may or may not lead to bankruptcy.
lead to a debt restructuring or a liquidation. The recovery rate may be greater or less depending on the particularities of the issuer.

Bankruptcy

- Bankruptcy is a *legal status* of an insolvent person or an organization that cannot repay the debts
- Imposed by a court order
- Most often initiated by a debtor.

Likelihood of Default

The likelihood of cash flow insolvency is increasing in

- Cash flow volatility of the issuer, and
- Coverage ratio (e.g., per period debt payments/ available cash)
- Covenants - creditors such as banks may declare a company bankrupt when

where “available cash” is, for example, measured by liquid assets + cash flow.

Recovery ratio

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In reality, recovery ratios average around 50%.

Figure 1. Recovery Rates and the Business Cycle

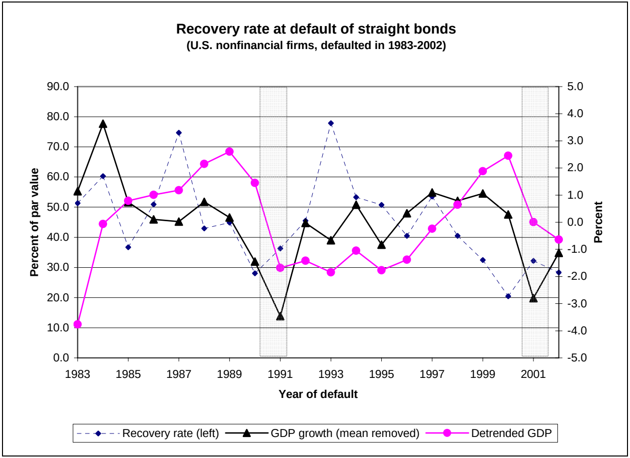

Figure: Historical recovery rates and GDP growth. From the paper "An Empirical Analysis of Bond Recovery Rates: Exploring a Structural View of Default" by Daniel Covitz and Song Han, The Federal Reserve Board

Let's start with a simple exercise. Suppose there is a constant probability p per period of default, $1 - p$ of survival default. Suppose the investor receives the cash flow C_t when the issuer survives, and X when it defaults. In the case of default the future cash flows cease. For simplicity, take the principal to be \$100 so that X is equal to the recovery rate.

Let y denote the ytm of a *non-defaultable* bond with the same characteristics.

Assume annual coupon, c.

The bond matures at date T.

Let V_t denote the value of the bond at time t.

Let p be the *risk neutral* probability of default.

At date T it is worth

$$V_T = (1 - p)(100 + c) + pX$$

We now find the value using the same approach as in with a tree: The value of continuation (non-default) at time $T - 1$ is $V_T/(1 + y)$ so that the total value (including coupon) is $V_T/(1 + y)$. If default we get X.

Thus, at date $T-1$ it is worth

$$V_{T-1} = (1-p)\frac{V_T}{1+y} + (1-p)c + pX$$

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$$
\begin{aligned}
V_{T-1} &= (1-p)\frac{V_T}{1+y} + (1-p)c + pX \\
&= (1-p)\frac{(1-p)(100+c) + pX}{1+y} + (1-p)c + pX
\end{aligned}
$$

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$$\begin{aligned} V_{T-1} &= (1-p)\frac{V_T}{1+y} + (1-p)c + pX \\ &= (1-p)\frac{(1-p)(100+c) + pX}{1+y} + (1-p)c + pX \\ &= \frac{(1-p)^2}{1+y}100 + \frac{(1-p)^2}{1+y}c + \frac{(1-p)pX}{1+y} + (1-p)c + pX \end{aligned}$$

Then again at date $T-2$

$$\begin{aligned} V_{T-2} &= \frac{(1-p)^3}{(1+y)^2}100 + \frac{(1-p)^3}{(1+y)^2}c + \frac{(1-p)^2}{(1+y)}c + (1-p)c \\ &+ \frac{(1-p)^2 pX}{(1+y)^2} + \frac{(1-p)pX}{(1+y)} + pX \end{aligned}$$

and so on.

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&\quad + \frac{(1-p)^2 pX}{(1+y)^2} + \frac{(1-p)pX}{(1+y)} + pX
\end{aligned}
$$

and so on. We deduce that the value is

$$
B_0 = (1-p)\left(100\frac{(1-p)^T}{(1+y)^T} + \sum_{t=1}^{T}\frac{(1-p)^t}{(1+y)^t}c\right) + \sum_{t=0}^{T}\frac{(1-p)^t}{(1+y)^t}pX \quad (1)
$$

Let's define an default adjusted discount rate

$$\frac{1}{1+y^*} = \frac{1-p}{1+y}$$

or

$$y^* = \frac{1+y}{1-p} - 1$$

For example, if $y = 0.01$ and the default probability is $p = 0.01$ then

$$y^* = 0.0202$$

or with $y = 0.02$ and $p = 0.04$

$$y^* = 0.0625$$

etc.
We see that $y^* \approx y + p$.

We can rewrite eqn. (1) as

$$B_0 = (1-p)\left(100\frac{1}{(1+y^*)^T} + \sum_{t=1}^{T}\frac{1}{(1+y^*)^t}c\right) + p\sum_{t=0}^{T}\frac{1}{(1+y^*)^t}X \quad (2)$$

In other words, we can discount the bond's cash flows using the "default adjusted ytm" y^* and probability weigh the value with and without default.

The formula in (3) can be written

$$\begin{aligned} B_0 = (1-p) \times & \text{ NPV of bond discounted at } y^* \text{ given no default} \\ & + p \times \text{NPV of recovery value discounted at } y^* \end{aligned}$$

Note: If we assume that the bond cannot default at time 0, then the value is

$$B_0 = \frac{100}{(1+y^*)^T} + \sum_{t=1}^{T} \frac{1}{(1+y^*)^t} c + \frac{p}{1-p} \sum_{t=1}^{T} \frac{1}{(1+y^*)^t} X \quad (3)$$

We can also use that

$$\sum_{t=1}^{T} \frac{1}{(1+y^*)^t} = \frac{1-(1+y^*)^{-T}}{y^*} =: A(y^*, T) \qquad (4)$$

is the annuity formula and write

$$B_0 = \frac{100}{(1+y^*)^T} + A(y^*, T)c + \frac{p}{1-p} A(y^*, T)X \qquad (5)$$

Assuming that the bond cannot default at $t = 0$.

- If the bond has zero recovery, then its value is just the first term. In this case, the "default adjusted discount rate", y^*, is the ytm on the bond assuming no default.
- Accordingly, with zero recovery, the spread on a defaultable bond is approximately equal to the (risk neutral) probability of default.

Example

- On April 6th a Yahoo finance reports that a 20 year AAA and A rated corporate bonds are yielding 5.31% and 5.48% respectively.
- Let's assume these are new issues (so they trade at par)
- Compare to a Treasury with 2.85% ytm
- What are the default probabilities?

First off, assuming zero recovery, we find that the implied default probability will solve

$$1 + y^* = 1.0531 = 1.0285/(1 - p)$$

for the AAA. So

$$p = 1 - \frac{1.0285}{1.0531} = 0.0234$$

is the implied default probability for the AAA.

However, the zero recovery rate assumption is extreme. Most recovery rates are higher.
With a 60% recovery the implied default probability is

$$p = 0.0542$$

We find this by solving eqn. (5) for p so that the bond coupon equals the stated 5.31% and the bond is selling at par.
Similarly we find

$$p = 0.0578$$

for the single A rated group.

Remarks

- Our model is an example of so-called *reduced form models*
- Shortcoming with our model: Constant default probability assumption
- More general reduced form models use time-varying default probabilities

Heuristics

- Suppose two periods. Invest at time 0. Firm pays coupon and principal at time 1, or it defaults and we get X.
- The firm needs to pay a premium $p(100 - X)$ to compensate investors for potential default losses.
- Thus, if you observe a corporate bond yield Y percent above the similarly dated treasury, the implied default probability is $Y100/X$. So for example if we observe a bond yielding a ytm of 5%, the treasury being 3% (so the spread is 2%), the implied default probability is approximately $100 \times 0.02/50 = 4\%$.

Heuristics II

- See that the heuristics from the one period example holds approximately in the graph.
- For example, in the above example $p = 0.0364$.
- Graph on next page gives more details

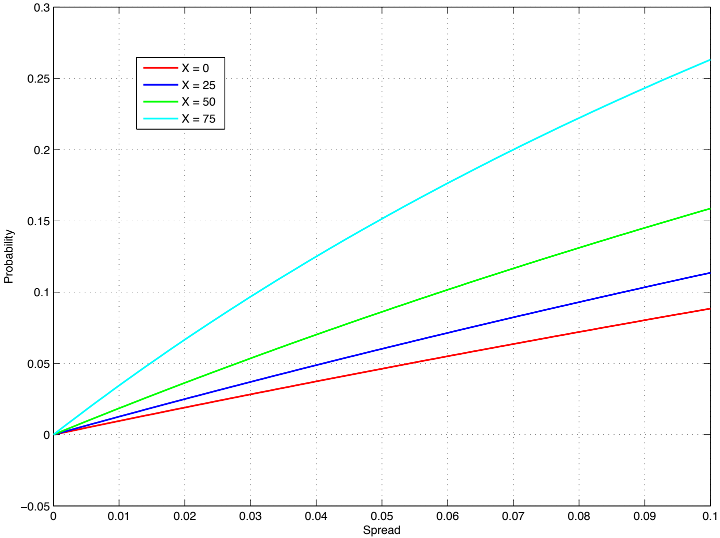


Figure: Default probability (risk neutral) implied by spreads.

Structural Models

- So-called structural models start with modeling firm values.
- Default is triggered by total firm value dropping below a threshold
- Black & Sholes (1972) option pricing formula as well as Merton's (1974) models are the classic models

Black & Sholes' insight

- Think of the risky bond as a portfolio of a risk free bond + a short position in a put option
- Underlying: Total firm value
- Strike = default boundry
- Put option is in the money (firm defaults) when total firm value is below the default boundary ("strike")

The value of a corporate bond is therefore

$$\begin{aligned} \text{risky bond value} &= \text{non-risky bond value} \\ &- \text{ value of put option on firm's total assets} \end{aligned}$$

Numerical example

Total assets = 100, debt (book value) = 50. Equity volatility is 30%. Yield curve is flat at 3%. Since the equity is levered, we

back out the total asset volatility using that

$$\text{Std(equity returns)} = L\text{Std(total asset returns)} \tag{6}$$

where L is the leverage ratio.
So the total asset return volatility is $30\% \times \frac{1}{2} = 15\%$.

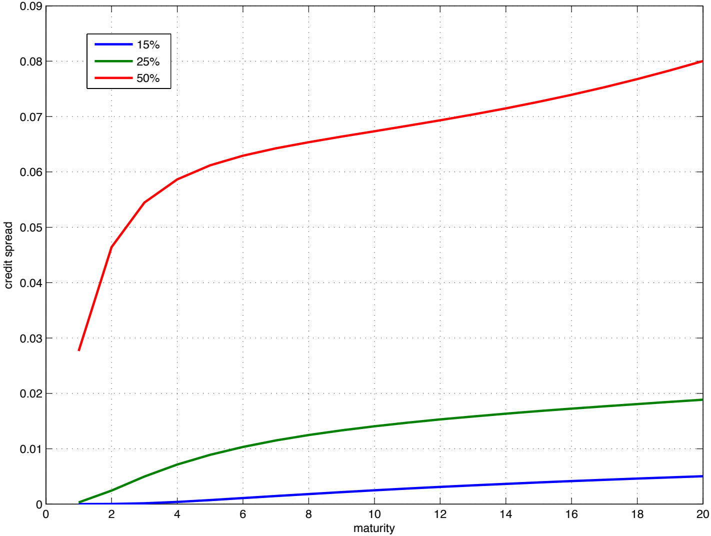
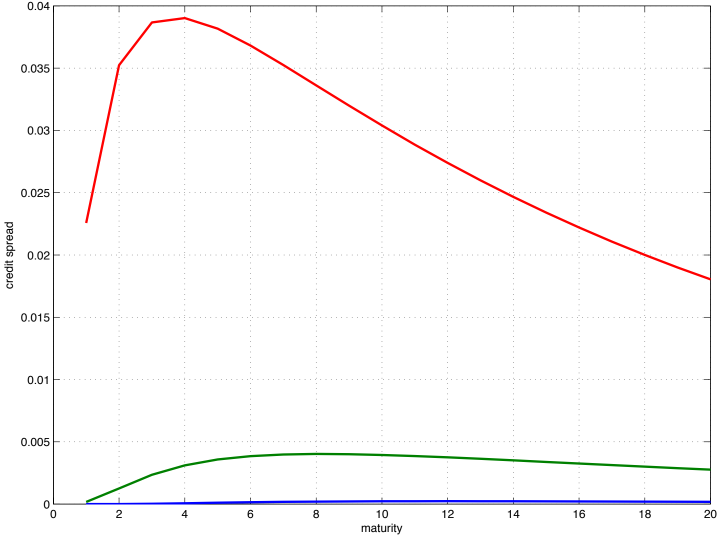


Figure: Credit spreads generated by the Black-Scholes model for different levels of total asset return volatility. Treasury curve flat at 0.



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Remarks

- Need really high equity vol to generate high spreads
- BS model only allows default at maturity (shortcoming)
- Lower spreads for short maturity debt - this is unique to the structural approach
- Spreads *decrease* when maturity increases and rates are positive (second graph)

Generalization

- Default any time
- Generalized dynamics to total asset values (jumps, stochastic volatility, etc

Important to understand:

- Long position's in corporate bonds are equivalent to long positions in treasuries + short position in equity options
- Therefore: when purchasing a corp bond you are *selling volatility*