# Basic Bond Stuff

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# Present Values

In basic finance, the *present value* of a cash flow stream $C_t$ paid at times $t = 1, .., T$ is

$$PV = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}. \tag{1}$$

In fixed income, all market prices obtain as the present value of the future risk free cash flows.

- The formula assumes that interest (coupon) is paid only at integer future times (i.e., in exactly 1, 2, ... years from now)
- The formula assumes that there is some common interest rate, $r$, that can be used to discount all the future cash flows.

In reality, either assumption is too restrictive.

We need

- To be able to discount using semi-annual coupons
- To be able to discount when the time to the first coupon is not an integer. In a typical case there might be 22/360 years to the first coupon and (181+22)/360 to the second, etc.
- To find a way to discount cash flows without assuming that we can use the same discount rate for all future cash flows

# Semi-Annual Coupons

If a bond pays coupon $C$ twice a year, it means that with approximately six month intervals the bond holder is paid $C/2$. Let $C_i$ denote the semi-annual cash flow from the bond. For example, a 6% coupon with maturity 2 years pays

Table: 1. Cash flows for a Semi-annual 6% w/ 2 yrs expiration

| | | | | |
|---|---|---|---|---|
| year | 1/2 | 1 | 1 1/2 | 2 |
| $C_i$ | 3 | 3 | 3 | 103 |

Here $C_i$ represents the $i$th cash flow, so with semi-annual coupons, the index will run from $i = 1, .., 4$ ($T \times 2$).

Now suppose $r$ is an annual discount rate. The present value would be

$$PV = \sum_{i=1}^{2T} \frac{C_i}{(1 + r/2)^i}$$

More generally, if cash flows occur $n$ times per year the present value is

$$PV = \sum_{i=1}^{nT} \frac{C_i}{(1 + r/n)^i}$$

# n days to the first coupon

In general, bonds do not have 1/2 or 1, or 4 years until their future coupon payments. Suppose for example that a bond has the following payment structure

Table: 2. Cash flows for a Semi-annual 6% w/ 568 day expiration

| | | | | |
|---|---|---|---|---|
| days | 22 | 204 | 387 | 568 |
| $C_i$ | 3 | 3 | 3 | 103 |

We can now compute the present value assuming an annual discount rate $r$, as

$$PV = \frac{3}{(1+r/2)^{\frac{22}{182.5}}} + \frac{3}{(1+r/2)^{\frac{204}{182.5}}} + \frac{3}{(1+r/2)^{\frac{387}{182.5}}} + \frac{103}{(1+r/2)^{\frac{586}{182.5}}}. \quad (2)$$

This formula can be generalized in the following way:

- Let $n$ be the number of future cash flows (here $n = 4$)
- Let $t_1, t_2, .., t_n$ be the number of days between the settlement date and cash flows $1, 2, .., n$

then

$$PV = \sum_{i=1}^{n} \frac{C_i}{(1+r/2)^{\frac{t_i}{182.5}}} \quad (3)$$

# Yield to maturity (ytm)

Let $P$ be the current price of a bond. The yield-to-maturity is the value of the discount rate, $y$, that solves

$$P = \sum_{i=1}^{n} \frac{C_i}{(1 + y/2)^{\frac{t_i}{182.5}}}. \tag{4}$$

- The ytm is an *internal rate of return* on the bond investment.
- The ytm must generally be found numerically through trial and error

# ytm of zero coupon bonds

Consider bonds that pay no coupon (zero coupon bond). In this case we can find the ytm easily. For a zero with maturity in $T$ days and $ 1 face value, the ytm can be found by solving

$$P = (1 + y/2)^{-T/182.5}$$

which has solution

$$y = 2(P^{-182.5/T} - 1).$$

# Interpreting ytm's

- The YTM should be interpreted as the rate of return earned by an investor who holds the bond *until maturity*.
- Note: Tuckman makes an argument against this interpretation based on the fact that one typically cannot reinvest coupons at a rate equal to the ytm. This argument relies on a too narrow definition of a return.
- You can also think of the ytm as the *approximate* expected rate of return going forward. This is the idea behind "riding the yield curve" trades (see later)

# The Discount Function

We generally cannot discount cash flows with different maturities using the same discount factor.

The *discount function* is a construction that allows us to find the (market) value of any non-risky claim at some future date $t$.

Specifically, the value of one dollar paid at time $t$ is

$$d(t).$$

Accordingly, the value of a coupon bond with cash flows $C(t_i)$, $t_i, i = 1, .., n$ is

$$\text{mkt value} = \sum_{i=1}^{n} C(t_i) d(t_i).$$

We will spend significant amount of time discussing how to compute the discount function and we will introduce *three* different methods for doing so....

# The term structure

The term structure of interest rates is the collection of ytm's for bonds of different maturity.

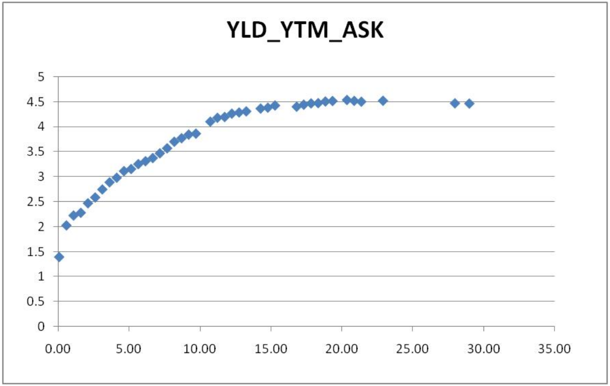


Figure: Coupon Yield Curve on Jul 15, 2008.

- Preceding graph was created from all non-callable Bills, notes, and bond asking prices.
- The code "YLD_YTM_ASK" is the Bloomberg identifier. These are YTM's computed from asking price by Bloomberg. The data were imported directly into Excel
- Note the near continuous feature of the yields. We can fit a non-linear function through the data with very small errors (we won't because it could give misleading insights...)
- The Jul 15, 2008 yield curve is typical as it is (severely) *upward sloping* - the short term yields are lower than the long ones

# Zero coupon yields

- Coupon yld curve is really an apples to oranges comparison because coupon bonds may have very different coupon rates. For example, there is no reason why a 5 year 10% coupon should have the same YTM as a 1% coupon.
- The zero coupon yield curve is particularly important because we can use it to price all other bonds in the economy
- We will discuss three different ways to derive zero coupon yield curves.

# Why the zero coupon curve is so important...

Consider a 2 year 10% coupon. It pays $ 5 coupon in 6,12,18 months from now and $ 105 in 24 months.

If there exist zero coupon bonds with $ 1 principals that mature on the same date as the coupon payments on the above bonds, we can *replicate* the payoff on the coupon bond with a portfolio of 5 zeros of maturities 6,12,18 months and 105 with 24 month maturity.

Since the payoffs on the portfolio of zeros is the same as the coupon, the must have the same market price. Otherwise:

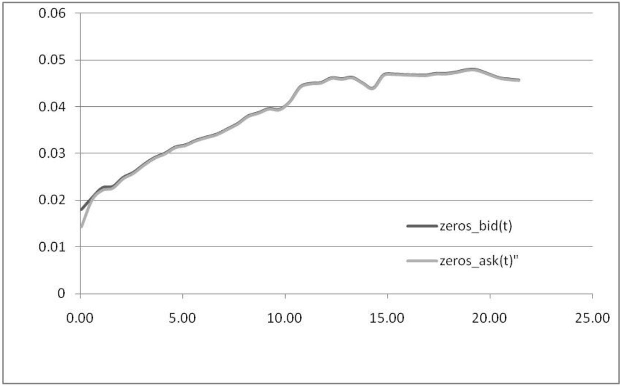


Figure: Zero Coupon Yield Curve on Jul 15, 2008.

Comments:

- The curve is *implicit* in coupon bonds (method for computing later..)
- The zero curve is more bumpy at 10-15 year horizons
- It dips at the end, suggesting that more recently issued 30 yr bonds have higher prices (lower yields) than past issues
- The graph shows the curve computed from both bids and asks. The small difference suggests tight b/a spreads.

# Fed Policy

- The Funds Rate is the rate at which the Fed Reserve borrow and lend in the interbank market.
- It is the Government's primary instrument for monetary policy
- The Federal Reserve controls the *Fed Fund target rate*.
- Fed fund target rate is determined at regular FOMC (Fed. Open Mkt. Committee) meetings every 6-7 weeks. Emergency meetings occur as well.
- The *Effective Fed funds rate* is the rate at which banks can borrow/ lend from the Fed and to each other short term (one day). It is market determined.
- Fed fund target rate is, as the name suggests, a *target* for the effective rate. The two are highly correlated ..

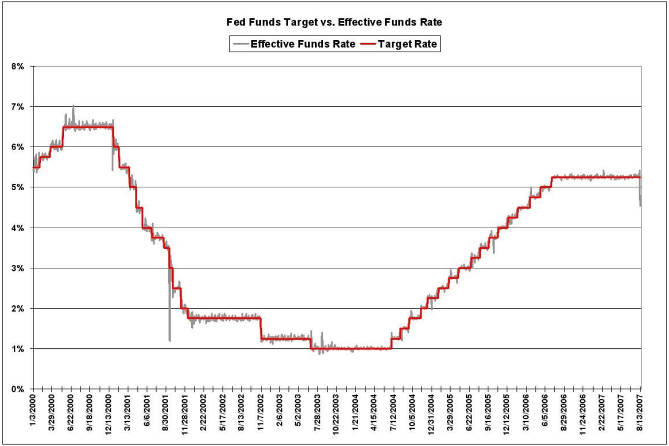


Figure: Effective vs target fed funds rates.


Figure: Effective vs target fed funds rates. 2010-2014.

# Fed Policy and the Yield Curve

Small deviations between effective and target rates due to temporary supply/ demand imbalances (micro market structure effects).

- The Fed determines the short end of the yield curve.
- Fed fund policy is dictated by various goals set by the particular Fed Reserve chairman. Typically in US, inflation targets and economic growth. Other countries may stabilize currency as well.
- The *Taylor Rule* of monetary policy dictates that the central bank should adjust rates according to target inflation and output (GDP growth)

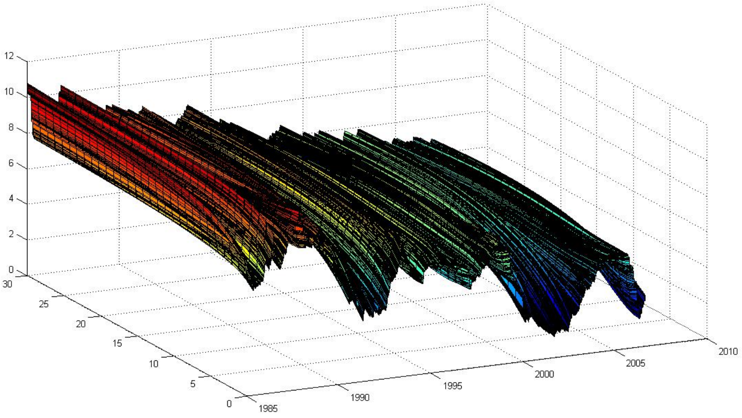


Figure: Daily zero coupon yields from Nov '85 to Aug '08.

Comments on yield curve:

- Long end of the yield curve moves much in parallel to the short end but short rates do NOT explain all the variation in long rates.
- Zero coupon yields are very highly correlated.
- typical shape: upward sloping
- Inverted yield curve ( when long rates exceed short ) occur occasionally.
- Note that rates have declined a lot since early 80ties

# A question to ponder...

Q: In December 1985, a 30 year zero coupon would sell at a ytm of 10.73%. Suppose you bought the bond then. Suppose you held this bond until the Fall of 2008. It would have 7 2/12 year maturity left and was selling at a price corresponding to a ytm of about 3.79%. What would be your return on this investment?

A: There are 22 10/12 years from the initial investment until now (Aug 1st, 08) and 7 2/12 left until expiration (note that the two add up to 30).
The price in 85 would be

$$P_{85} = 1.1073^{-30} = 0.047$$

while the 08 price is

$$P_{08} = 1.0379^{-7.16667} = 0.766.$$

A dollar invested in '85 would have grown to

$$0.766/0.047 = 16.2996$$

today.
The average annual rate of return is

$$\left(\frac{P_{08}}{P_{85}}\right)^{\frac{1}{22.83333}} - 1 = 0.13$$

Note that the average return exceeds the ytm. This is because rates *went down* over the period.
Suppose instead that the ytm had remained unchanged at 10.73%. Then the average return would have been 10.73%.